Chapter 4

Linear Equations in One Unknown

Learning Objectives

After completing this chapter, you will be able to

- master the techniques of solving linear equations in one unknown.
- get answers to daily-life problems by setting up and solving linear equations in one unknown.

A sheet of paper is cut into 3 pieces. One of them is then cut into another 3 pieces. The same process goes on with the same method. Can we get exactly 30 pieces of paper?

Preview

[Basic knowledge and techniques required for this chapter.]

A. Basic Knowledge

Using algebraic expressions to represent statements

Statement	Algebraic expression	Variable
The perimeter of a square of sides w m each	$4w$ m	w
The time required for Mandy to run d km at a speed of 5 km/h	$\frac{d}{5}$ h	d
The amount of Mike's *savings* after n days, if he has saved \$1 and starts saving \$2 every day	\$$(2n+1)$	n

B. Basic Technique

1. Addition and subtraction of directed numbers

For example:

(a) $(+7)+(+3)=7+3$
$\qquad =10$

(b) $(+8)+(-2)=8-2$
$\qquad =6$

(c) $(-4)+(+2)=-4+2$
$\qquad =-2$

(d) $(-3)+(-6)=-3-6$
$\qquad =-9$

(e) $(+7)-(+3)=7-3$
$\qquad =4$

(f) $(+8)-(-2)=8+2$
$\qquad =10$

(g) $(-4)-(+2)=-4-2$
$\qquad =-6$

(h) $(-3)-(-6)=-3+6$
$\qquad =3$

2. Multiplication and division of directed numbers

For example:

(a) $(+4)(+2)=8$

(b) $(+8)(-4)=-32$

(c) $(-10)(+5)=-50$

(d) $(-12)(-4)=48$

(e) $\frac{+4}{+2}=2$

(f) $\frac{+8}{-4}=-2$

(g) $\frac{-10}{+5}=-2$

(h) $\frac{-12}{-4}=3$

savings 儲蓄

4.1 Concept of Equation

Class Activity 4.1

Aim: To set up algebraic expressions with equal signs according to daily-life situations

Happy Restaurant serves dim sum every day. The price of a large dish is twice that of a small dish.

Tang's family ordered 3 small dishes and 2 large dishes. Together with a $10 charge for the tea, they paid a total of $94.

1. Let the price of a small dish be $\$x$. Complete the following table by using algebraic expressions of x.

Show Ans.

	Amount ($)
Price of a small dish	x
Price of a large dish	
Total price of 3 small dishes, 2 large dishes and a $10 charge for the tea	

2. (a) Use an equality of x to show that 'the total price of 3 small dishes, 2 large dishes and a $10 charge for the tea is $94'.

Show Ans.

________________ = ________________

(b) What value of x satisfies the equality obtained in **2(a)**?

Hence,
the price of a small dish is $________,
the price of a large dish is $________.

Now I see ...
Using a letter to represent an unknown value in a question can help to form an equality. This equality, in turn, can help to find the unknown value.

When a letter x is used to represent the unknown value in a question, x is called the **unknown**. An equality which contains an unknown is called an **equation**.

e.g. $x+2=9$, $3y-1=5$

If an equation has one unknown only and the unknown is of degree 1 (such as the two examples above), the equation is called a **linear equation in one unknown**.

> If two sides of an equation are equal after substituting a value for the unknown of the equation, the value is called the **solution** (or **root**) of the equation.

e.g. For the equation $3x+2(2x)+10=94$,

substitute $x=12$ into the equation,

L.H.S. $=3(12)+2[2(12)]+10=36+48+10=94$

R.H.S. $=94$

$\therefore$ $x=12$ is the solution of the equation $3x+2(2x)+10=94$.

'L.H.S.' means 'left-hand side'.

'R.H.S.' means 'right-hand side'.

> To solve an equation is to find its solution.

Do you remember the methods of solving equations learned from primary education? Use them to complete Extension 4.1.

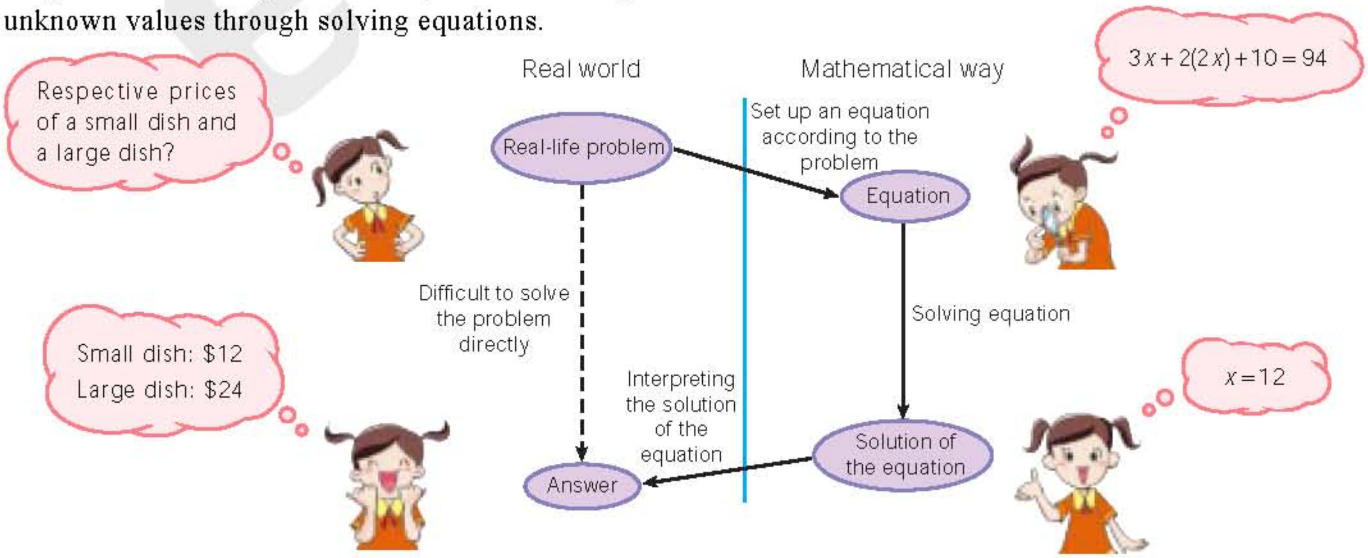
Extension 4.1

Solution

Solve the following equations.

(a) $w+2=6$ (b) $x-9=2$ (c) $5y=75$ (d) $\frac{z}{3}=7$

Very often, we use equations to represent real-life problems and find the unknown values through solving equations.



unknown 未知數 equation 方程 linear equation in one unknown 一元一次方程 solution 解 root 根

In the next section, we will learn some techniques of solving linear equations in one unknown. Following that, we will learn to solve problems by using equations.

Class Activity 4.2

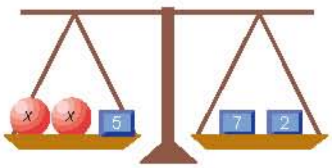
Aim: To explore the way of solving equations

Equation $2x+5=9$ can be represented by a beam-balance in a balanced position as follows. Same action is applied to each side of the beam-balance. Describe the action and write down the corresponding equation.

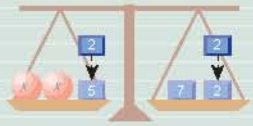
Show Ans.

Action applied to both sides

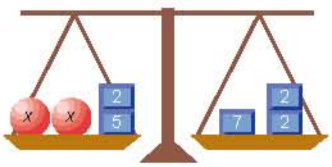
① Equation: $2x+5=9$



Adding 2



② Equation: ________



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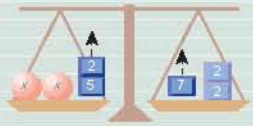
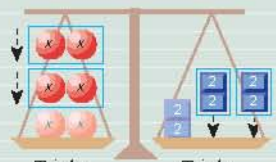


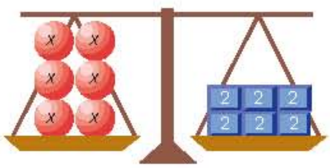
③ Equation: ________

____



Triple Triple

④ Equation: ________



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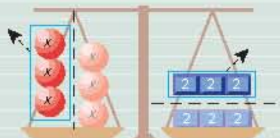
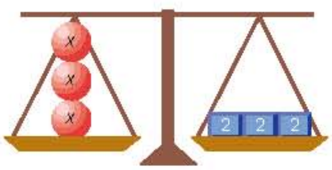


Reduced to half Reduced to half

⑤ Equation: ________



Both sides of the beam-balance keep in a balanced position when the same action is applied to them.

Both sides of the equation remain ____________ when the same operation is applied to them.

Now I see ...

Both sides of the equation remain equal when the same operation is applied to them.

I.T. Tool

Student's CD

In Class Activity 4.2, even though the equation keeps changing, both sides of it remain equal.

① → ②: Add 2 to both sides of the equation.

② → ③: Subtract 7 from both sides of the equation.

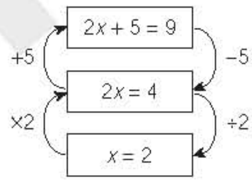
③ → ④: Multiply both sides of the equation by 3.

④ → ⑤: Divide both sides of the equation by 2.

> When solving an equation, the unknown can be found by applying the same operation to both sides of the equation.

e.g. the steps of solving equation $2x+5=9$ are as follows:

$$
\begin{aligned}
2x+5&=9\\
2x+5-5&=9-5 && \text{◂ Subtract 5 from both sides.}\\
2x&=4\\
\frac{2x}{2}&=\frac{4}{2} && \text{◂ Divide both sides by 2.}\\
x&=2
\end{aligned}
$$



Example 4.1 Solving simple equations

Additional Example 4.1

Solve the following equations.

(a) $2x-3=11$

(b) $25-4x=13$

(c) $\dfrac{x}{2}+5=8$

(d) $\dfrac{10+2x}{3}=18$

Solution

(a)

$$
\begin{aligned}
2x-3&=11\\
2x-3+3&=11+3 && \text{◂ Add 3 to both sides.}\\
2x&=14\\
\frac{2x}{2}&=\frac{14}{2} && \text{◂ Divide both sides by 2.}\\
x&=\underline{\underline{7}}
\end{aligned}
$$

Classwork 4.1

Solution

Solve the following equations.

(a) $3x+2=23$

(b) $29-3x=11$

(c) $31-\dfrac{3x}{4}=19$

(d) $\dfrac{x-3}{4}=2$

(b)

$$
\begin{aligned}
25 - 4x &= 13 \\
25 - 4x - 25 &= 13 - 25 && \blacktriangleleft \text{ Subtract 25 from both sides.} \\
-4x &= -12 \\
\frac{-4x}{-4} &= \frac{-12}{-4} && \blacktriangleleft \text{ Divide both sides by } -4. \\
x &= \underline{\underline{3}}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\frac{x}{2} + 5 &= 8 \\
\frac{x}{2} + 5 - 5 &= 8 - 5 && \blacktriangleleft \text{ Subtract 5 from both sides.} \\
\frac{x}{2} &= 3 \\
\frac{x}{2} \times 2 &= 3 \times 2 && \blacktriangleleft \text{ Multiply both sides by 2.} \\
x &= \underline{\underline{6}}
\end{aligned}
$$

(d)

$$
\begin{aligned}
\frac{10 + 2x}{3} &= 18 \\
\frac{10 + 2x}{3} \times 3 &= 18 \times 3 && \blacktriangleleft \text{ Multiply both sides by 3.} \\
10 + 2x &= 54 \\
10 + 2x - 10 &= 54 - 10 && \blacktriangleleft \text{ Subtract 10 from both sides.} \\
2x &= 44 \\
\frac{2x}{2} &= \frac{44}{2} && \blacktriangleleft \text{ Divide both sides by 2.} \\
x &= \underline{\underline{22}}
\end{aligned}
$$

Skills Upgrading Corner 4.1

Solution

Solve the following equations.

(a) $2x + 13 = 7$

(b) $\frac{x}{3} - 2 = 5$

(c) $14 - 5x = -11$

(d) $4 + \frac{x}{6} = 0$

(e) $11 - \frac{3x}{5} = 2$

(f) $\frac{2x - 9}{3} = 1$

Exercise 4A

Level 1

Solve the following equations. (**1** – **6**)

1. **(a)** $2x-12=6$ **(b)** $13-5x=-2$
(c) $6x+1=7$ **(d)** $3=15+2x$

2. **(a)** $5+4y=9$ **(b)** $19-2y=11$
(c) $14=8-3y$ **(d)** $5=14+3y$

3. **(a)** $\frac{z}{8}-1=1$ **(b)** $\frac{z}{2}+6=5$
(c) $9-\frac{z}{2}=14$ **(d)** $5=\frac{z}{11}+7$

4. **(a)** $\frac{2p}{3}=4$ **(b)** $\frac{3p}{2}=-12$
(c) $\frac{4p}{5}=8$ **(d)** $\frac{3p}{14}=\frac{6}{7}$

5. **(a)** $\frac{5+q}{6}=9$ **(b)** $\frac{2-q}{3}=7$
(c) $\frac{q+4}{5}=8$ **(d)** $\frac{q-8}{2}=-15$

6. **(a)** $2-\frac{6r}{5}=8$ **(b)** $14+\frac{10}{7}r=4$
(c) $6-\frac{2}{3}r=2$ **(d)** $15+\frac{7r}{15}=8$

Level 2

Solve the following equations. (**7** – **11**)

7. **(a)** $\frac{4y-3}{3}=7$ **(b)** $-5=\frac{7-3y}{4}$

8. **(a)** $\frac{2}{3}(a-2)=6$ **(b)** $\frac{1}{5}(2a-3)=3$

9. **(a)** $\frac{-(6-8b)}{7}=6$ **(b)** $\frac{-3b-12}{9}=2$

10. **(a)** $\frac{1}{2}-\frac{x-3}{4}=0$ **(b)** $\frac{7-x}{2}+2=5$

11. **(a)** $\frac{3y-5}{2}-6=5$ **(b)** $10-\frac{4y+9}{3}=3$

4.2 Techniques of Solving Equations

A Combining unknowns

Sometimes, we may need to combine unknowns to solve an equation.
For example,

(a) $$\begin{aligned} 5x + 2x &= (x + x + x + x + x) + (x + x) \\ &= x + x + x + x + x + x + x \\ &= 7x \end{aligned}$$

(b) $$\begin{aligned} 5x - 2x &= (x + x + x + x + x) - (x + x) \\ &= x + x + x + x + x - x - x \\ &= 3x \end{aligned}$$

Additional Example 4.2

Example 4.2 Combining unknowns to solve equations

Solve the following equations.

(a) $5x - 2x = 21$

(b) $4x - 6 = x$

(c) $6x + 5 = 21 - 2x$

Classwork 4.2 Solution

Solve the following equations.

(a) $6x - 3x = 12$

(b) $3x + 14 = 10x$

(c) $6x - 9 = 2x + 11$

Solution

(a) $$\begin{aligned} 5x - 2x &= 21 \\ 3x &= 21 && \text{Combine } x. \\ x &= \frac{21}{3} && \text{Divide both sides by 3.} \\ &= \underline{\underline{7}} \end{aligned}$$

(b) $$\begin{aligned} 4x - 6 &= x \\ 4x &= x + 6 && \text{Add 6 to both sides.} \\ 4x - x &= 6 && \text{Subtract } x \text{ from both sides.} \\ 3x &= 6 \\ x &= \frac{6}{3} \\ &= \underline{\underline{2}} \end{aligned}$$

(c) $$\begin{aligned} 6x + 5 &= 21 - 2x \\ 6x + 2x + 5 &= 21 && \text{Add } 2x \text{ to both sides.} \\ 8x + 5 &= 21 \\ 8x &= 21 - 5 \\ 8x &= 16 \\ x &= \frac{16}{8} \\ &= \underline{\underline{2}} \end{aligned}$$

B Removing brackets

Consider $2\times(3+5)=(3+5)+(3+5)$
$=3+3+5+5$
$=2\times3+2\times5$

i.e. $2\times(3+5)=2\times3+2\times5$

Similarly, $3(2a-b)=(2a-b)+(2a-b)+(2a-b)$
$=2a+2a+2a-b-b-b$
$=3(2a)-3b$

i.e. $3(2a-b)=3(2a)-3b$

In general,

$a(b+c)=ab+ac$

Also, we found that

$(b+c)a=ba+ca$

$a(b-c)=ab-ac$

$(b-c)a=ba-ca$

The above techniques help us to remove brackets in an equation.

Additional Example 4.3

Example 4.3 Removing brackets to solve equations

Solve the following equations.

(a) $5(x-2)=2x+8$

(b) $3(x-7)-4(x-10)=12$

(c) $7(2x-5)=4+3(3x-8)$

Solution

(a)

$$
\begin{aligned}
5(x-2)&=2x+8\\
5(x)-5(2)&=2x+8 && \text{Remove brackets.}\\
5x-10&=2x+8\\
5x-2x&=8+10 && \text{Subtract } 2x \text{ from both sides and add 10 to both sides.}\\
3x&=18\\
x&=\frac{18}{3}\\
&=\underline{\underline{6}}
\end{aligned}
$$

Classwork 4.3

Solution

Solve the following equations.

(a) $3(x+2)=4x-2$

(b) $9(x-2)-11(x-3)=3$

(c) $7(x-10)=4(x-9)-7$

(b)
$$\begin{aligned}
3(x-7)-4(x-10)&=12\\
(3x-21)-(4x-40)&=12\\
3x-21-4x+40&=12 \qquad \blacktriangleleft\ -(4x-40)=-4x+40\\
-x+19&=12\\
-x&=12-19\\
-x&=-7\\
x&=\underline{\underline{7}}
\end{aligned}$$

(c)
$$\begin{aligned}
7(2x-5)&=4+3(3x-8)\\
14x-35&=4+(9x-24)\\
14x-35&=4+9x-24\\
14x-35&=-20+9x\\
14x-9x&=-20+35\\
5x&=15\\
x&=\frac{15}{5}\\
&=\underline{\underline{3}}
\end{aligned}$$

C Technique of *eliminating* denominators

For equations with unknowns on both sides and with denominators, such as $\frac{1}{2}(x+3)=12-x$, $\frac{x+1}{4}=\frac{x-2}{3}$ and $\frac{2x}{3}-2=\frac{2x+10}{7}$, we may multiply both sides of an equation by the same value for further simplification.

Additional Example 4.4

Example 4.4 Solving equations by eliminating denominators

Solve the following equations.

(a) $\frac{1}{2}(x+3)=12-x$

(b) $\frac{x+1}{4}=\frac{x-2}{3}$

(c) $\frac{2x}{3}-2=\frac{2x+10}{7}$

Solution

(a)
$$\begin{aligned}
\frac{1}{2}(x+3)&=12-x\\
x+3&=2(12-x) \qquad \blacktriangleleft\ \text{Multiply both sides by 2.}\\
x+3&=24-2x\\
x+2x&=24-3\\
3x&=21\\
x&=\frac{21}{3}\\
&=\underline{\underline{7}}
\end{aligned}$$

Classwork 4.4

Solution

Solve the following equations.

(a) $6x+2=\frac{3x+10}{5}$

(b) $\frac{4x+15}{3}=\frac{21-x}{2}$

(c) $\frac{4x-10}{3}=\frac{15x}{8}-12$

eliminate 消去

(b)

$$\frac{x+1}{4}=\frac{x-2}{3}$$

$$12\times\frac{x+1}{4}=12\times\frac{x-2}{3}$$

∵ L.C.M. of 3 and 4 = 12
∴ Multiplying both sides by 12 can eliminate the denominators 3 and 4.

$$3(x+1)=4(x-2)$$
$$3x+3=4x-8$$
$$3x-4x=-8-3$$
$$-x=-11$$
$$x=\underline{\underline{11}}$$

(c)

$$\frac{2x}{3}-2=\frac{2x+10}{7}$$

$$21(\frac{2x}{3}-2)=21(\frac{2x+10}{7})$$

∵ L.C.M. of 3 and 7 = 21
∴ Multiplying both sides by 21 can eliminate the denominators 3 and 7.

$$21(\frac{2x}{3})-21\times 2=3(2x+10)$$
$$7(2x)-42=6x+30$$
$$14x-6x=30+42$$
$$8x=72$$
$$x=\frac{72}{8}$$
$$=\underline{\underline{9}}$$

Skills Upgrading Corner 4.2

Solve the following equations. (**1** – **4**)

Solution **1.** **(a)** $13x-15x=12$ **(b)** $-10x+4=-7x+13$ **(c)** $-3x+12=4x-9$

Solution **2.** **(a)** $-2(y-7)=7y-4$ **(b)** $4(y+3)=-8(y-6)$ **(c)** $-5(y-2)-9=3(y-13)$

Solution **3.** **(a)** $\frac{12-14z}{9}=15+3z$ **(b)** $\frac{-(7z+4)}{6}=\frac{26}{3}$ **(c)** $\frac{14-9z}{11}=\frac{z}{7}+8$

Solution **4.** **(a)** $\frac{p+13}{11}+\frac{p-2}{7}=3$ **(b)** $-\frac{p-4}{5}=3+\frac{p+11}{6}$ **(c)** $\frac{8p+15}{15}+\frac{4p-7}{9}=10$

Exercise 4B

Level 1

Solve the following equations. (**1** – **7**)

1. **(a)** $-3x-6x=36$ **(b)** $2x+3x=10$ **(c)** $5x-8x=15$

2. **(a)** $5y = 10y - 20$ **(b)** $4y + 6 = 6y$ **(c)** $-4y - 2y = -12$

3. **(a)** $5z + 7 = 3z - 11$ **(b)** $-11z + 4 = -8z + 28$ **(c)** $13 - z = 2z - 8$

4. **(a)** $3(x + 11) = 2x + 13$ **(b)** $-5(x + 10) = 5 - 10x$ **(c)** $6(2 - x) = -4x + 2$

5. **(a)** $\dfrac{p-2}{2} = p$ **(b)** $\dfrac{6-p}{3} = p + 6$ **(c)** $8 - p = \dfrac{p+16}{5}$

6. **(a)** $\dfrac{4q+2}{2} = q + 4$ **(b)** $10 - 2q = \dfrac{5q-2}{9}$ **(c)** $\dfrac{6-2q}{4} = 14 + 2q$

7. **(a)** $\dfrac{a+3}{3} = \dfrac{a-4}{4}$ **(b)** $\dfrac{10-a}{3} = \dfrac{a+10}{2}$ **(c)** $\dfrac{1-a}{8} = \dfrac{a-12}{3}$

Level 2

Solve the following equations. (**8** – **14**)

8. **(a)** $12(y - 9) = 6(y - 5)$ **(b)** $2(-9y - 15) = -4(5y + 10)$

9. **(a)** $5(z + 1) - 8(z - 3) = 11$ **(b)** $9(2z - 7) + 3(-2 - 12z) = -15$

10. **(a)** $\dfrac{3b+12}{9} = \dfrac{6b-2}{5}$ **(b)** $\dfrac{4b-5}{11} = \dfrac{2b+1}{5}$

11. **(a)** $\dfrac{-(7p-3)}{5} - 3p = 27$ **(b)** $4p + \dfrac{-3p-20}{4} = -5$

12. **(a)** $\dfrac{8r-10}{3} = \dfrac{7r}{4} + 4$ **(b)** $\dfrac{22-3r}{2} = \dfrac{r}{5} - 6$

13. **(a)** $\dfrac{m+5}{2} - \dfrac{m-6}{9} = 9$ **(b)** $\dfrac{-(m+17)}{8} + \dfrac{m-4}{5} = -3$

14. **(a)** $\dfrac{6x+8}{15} + \dfrac{5x-8}{6} = 14$ **(b)** $\dfrac{7x-3}{2} + \dfrac{3x-14}{8} = -11$

4.3 Solving Problems by Using Linear Equations in One Unknown

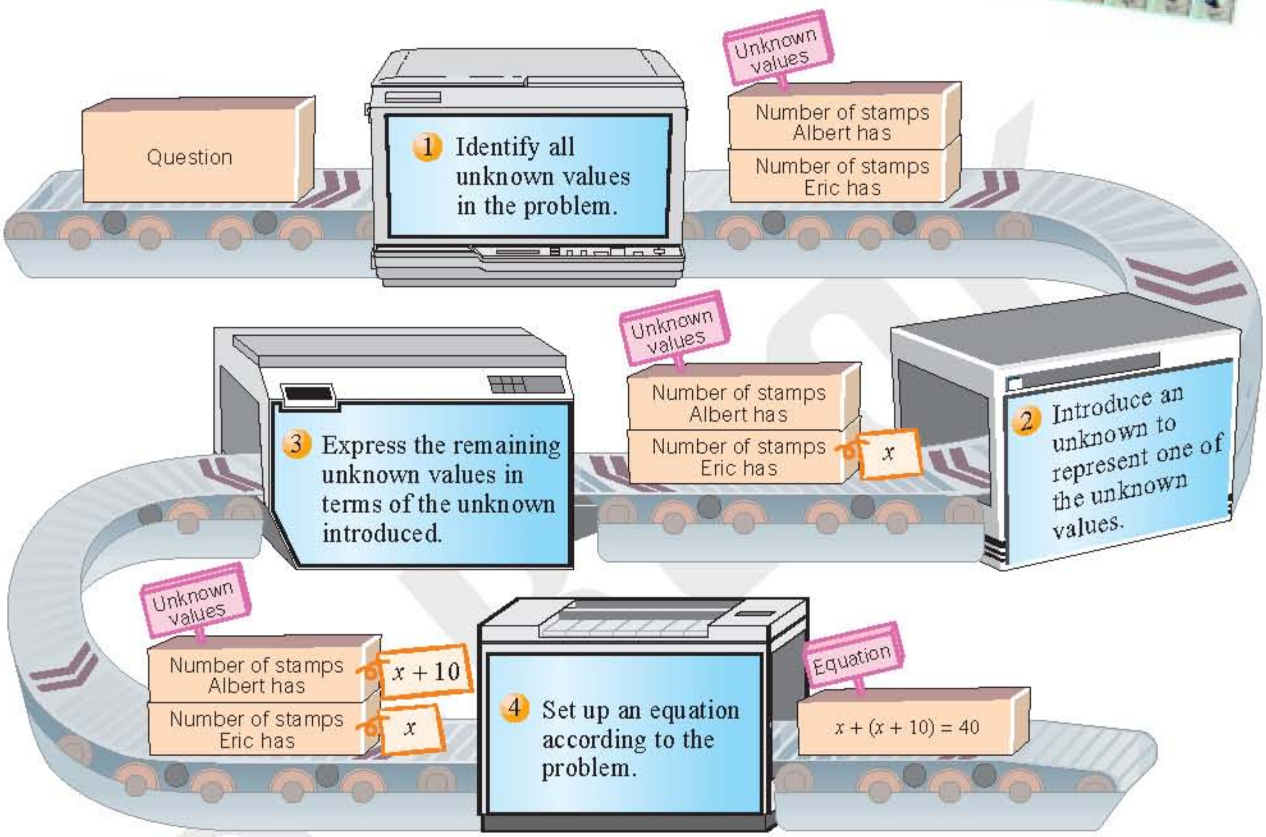
Using linear equations in one unknown to solve problems is common in daily life. In this section, we will learn how to turn a real-life problem into an equation and then get an answer by solving the equation.

A Setting up linear equations in one unknown

Consider the following problem:

'Albert has 10 stamps more than Eric does. Two of them have a total of 40 stamps. How many stamps does Eric have?'

Generally, a problem can be turned into an equation by following the simple steps below.



Additional Example 4.5

Example 4.5 Setting up simple equations

The number of apples owned by Melody is 6 times as many as Ken's. If Melody eats 5 apples, she will have 49 apples left. Set up an equation for finding the number of apples owned by Ken.

Classwork 4.5

Solution

In a party, the number of adults is 7 more than 9 times that of children. The total number of people there is 67. Set up an equation for finding the number of children.

Solution:

Let x be the number of children.

$\therefore$ The number of adults is

____________.

$\therefore$ The required equation is

____________.

Solution

[Analysis:

1. **Identify all unknown values:**
   Number of apples owned by Melody, number of apples owned by Ken.
2. **Introduce an unknown:**
   According to the problem, let a be the number of apples owned by Ken.
3. **Express the remaining unknown values by algebraic expressions:**

Number of apples owned by Ken	a
Number of apples owned by Melody	$6a$

6 times
After eating 5 apples, 49 apples left.

4. **Set up an equation: $6a-5=49$]**

Let a be the number of apples owned by Ken. ◂ 2

$\therefore$ Melody owns $6a$ apples.
After Melody has eaten 5 apples, $(6a-5)$ apples are left. 3

$\therefore$ The required equation is $\underline{\underline{6a-5=49}}$. ◂ 4

Additional Example 4.6

Example 4.6 Setting up an equation involving brackets

In a supermarket, the respective prices of a packet of lemon tea and green tea are \$4 and \$4.5. The total price of 17 packets of lemon tea and green tea is \$72. Set up an equation to find the number of packets of green tea.

Solution

[Analysis:

1. **Identify all unknown values:**
   Number of packets of lemon tea, number of packets of green tea, total price of lemon tea, total price of green tea.
2. **Introduce an unknown:**
   According to the problem, let x be the number of packets of green tea.
3. **Express the remaining unknown values by algebraic expressions:**

Number of packets of green tea	x
Number of packets of lemon tea	$17-x$
Total price of green tea	$\$4.5x$
Total price of lemon tea	$\$4(17-x)$

Selling price
Selling price
17 packets in total
\$72 in total

4. **Set up an equation: $4(17-x)+4.5x=72$]**

Let x be the number of packets of green tea. ◂ 2

$\therefore$ There are $(17-x)$ packets of lemon tea.
The total price of green tea is \$$4.5x$.
The total price of lemon tea is \$$4(17-x)$. 3

$\therefore$ The required equation is $\underline{\underline{4.5x+4(17-x)=72}}$. ◂ 4

Classwork 4.6

Solution

The prices of an adult ticket and a child ticket for an exhibition are as follows.

Adult: \$13 each
Child: \$9 each

A group of 9 persons spent \$97 on the tickets. Set up an equation to find the number of adults in the group.

Solution:

Let m be the number of adults.

$\therefore$ Number of children is ________________.

Total price of the adult tickets is \$________________.

Total price of the child tickets is \$________________.

$\therefore$ The required equation is ________________.

Additional Example 4.7

Example 4.7 Setting up an equation involving fractions

There are 120 oranges divided into 3 piles. The number of oranges in pile 1 is $\frac{1}{3}$ of that in pile 2, while the number of oranges in pile 2 is half of that in pile 3. Set up an equation to find the number of oranges in the biggest pile.

Classwork 4.7

Solution

Morris, Eason and Fred have a sum of money each. Morris has \$150 more than Eason does. Fred's amount is half of Morris' and is \$200 less than Eason's. Set up an equation to find the sum of money Eason has.

Solution

[Analysis: 1 Identify all unknown values:
Number of oranges in pile 1, number of oranges in pile 2, number of oranges in pile 3.

2 Introduce an unknown:
According to the problem, let x be the number of oranges in the biggest pile.

3 Express the remaining unknown values by algebraic expressions:

120 oranges in total

Number of oranges in pile 3	x	
Number of oranges in pile 2	$\frac{1}{2}x$	Half
Number of oranges in pile 1	$\frac{1}{3}(\frac{1}{2}x)=\frac{1}{6}x$	$\frac{1}{3}$

4 Set up an equation:

$\frac{1}{6}x+\frac{1}{2}x+x=120$]

Let x be the number of oranges in the biggest pile. ◂ 2

$\therefore$ There are $\frac{1}{2}x$ oranges in pile 2.

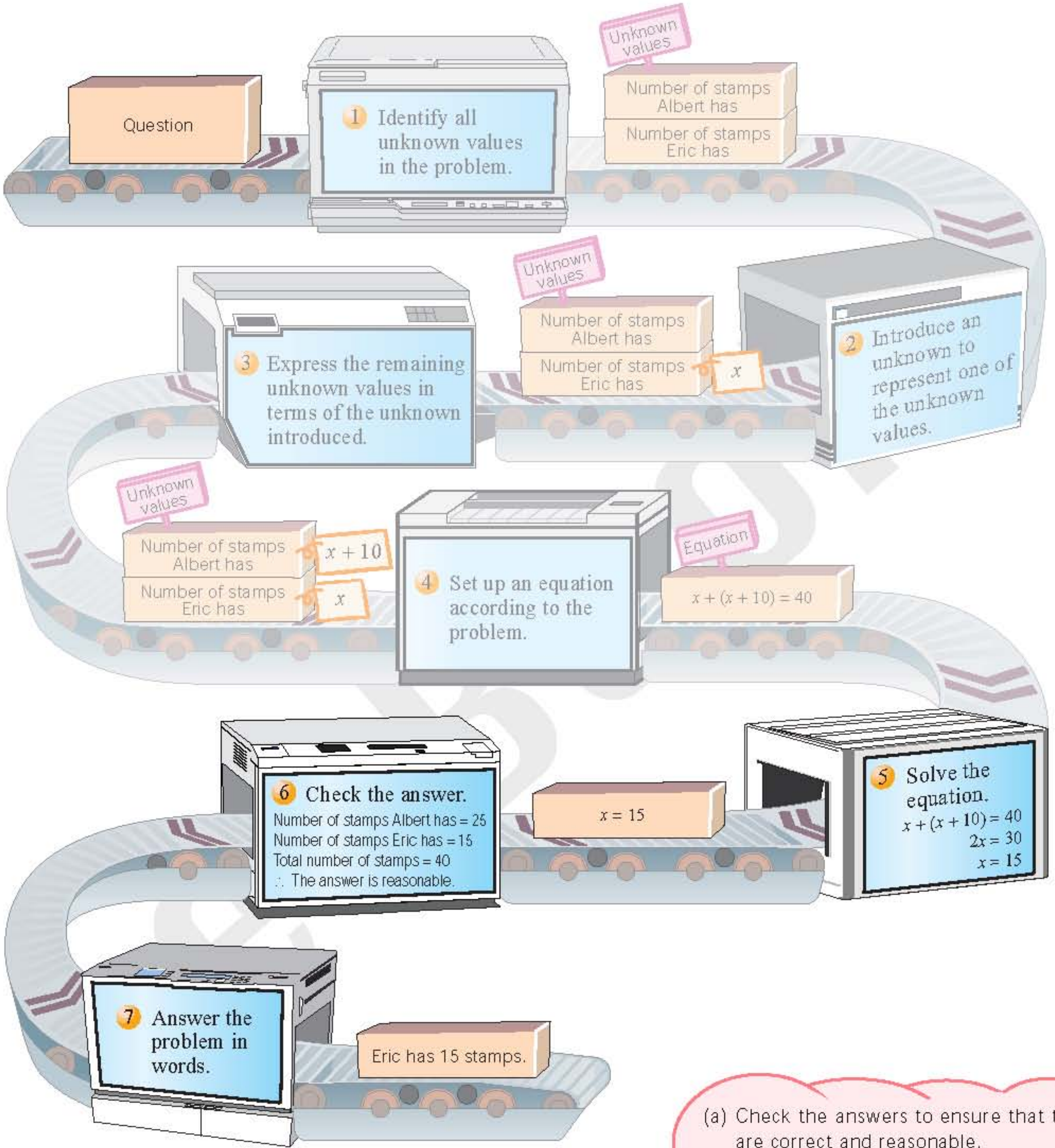
There are $\frac{1}{3}(\frac{1}{2}x)=\frac{1}{6}x$ oranges in pile 1. 3

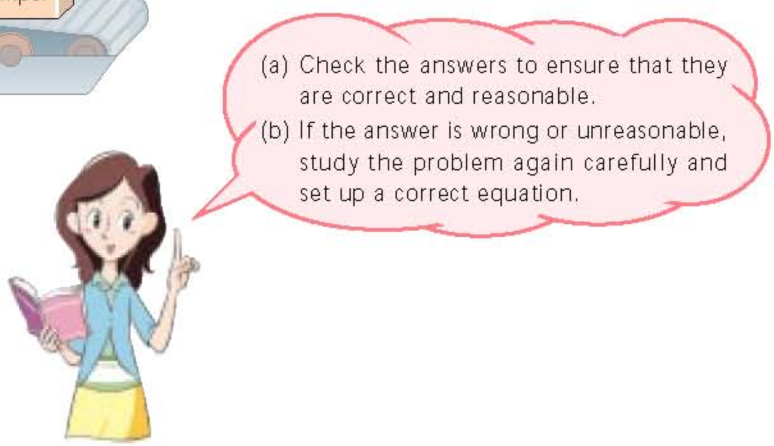
$\therefore$ The required equation is $\underline{\underline{\frac{1}{6}x+\frac{1}{2}x+x=120}}$. ◂ 4

B Solving problems by using equations

At the beginning of this section, we have used the example on the number of stamps Albert and Eric have to learn how to set up an equation. However, the problem has not yet been solved.

Generally, a problem can be solved by using an equation following the steps below.





Additional Example 4.8

Example 4.8 Solving problems by using equations

Nancy and Susan have a total of \$4 100. If Nancy gives \$200 to Susan, she will have 4 times as much as Susan has. How much does each of them have originally?

Solution

Let Susan have \$$p$ originally.

$\therefore$ Nancy has \$$(4\ 100 - p)$ originally.

If Nancy gives \$200 to Susan, Nancy will have \$$(4\ 100 - p - 200)$ and Susan will have \$$(p + 200)$.

$$\begin{aligned}
\therefore\quad 4\ 100 - p - 200 &= 4(p + 200)\\
3\ 900 - p &= 4p + 800\\
3\ 900 - 800 &= 4p + p\\
5p &= 3\ 100\\
p &= 620
\end{aligned}$$

$$\begin{aligned}
4\ 100 - p &= 4\ 100 - 620\\
&= 3\ 480
\end{aligned}$$

$\therefore$ Originally, Susan has \$620 and Nancy has \$3 480.

Checking:
If Nancy gives \$200 to Susan,
Nancy's amount $= \$(3\ 480 - 200)$
$= \$3\ 280$
Susan's amount $= \$(620 + 200)$
$= \$820$
$\therefore$ Nancy's amount is $\dfrac{3\ 280}{820} = 4$ times Susan's amount

Classwork 4.8

Solution

Mike has 4 times as many stamps as Kevin does. If Mike gives 16 stamps to Kevin, he still has 25 more stamps than Kevin does. How many stamps does each of them have originally?

Additional Example 4.9

Example 4.9 Solving problems by using equations

Elsa's age is $\frac{1}{7}$ of her mother's. After three years, the sum of their ages will be 46. Find the current age of Elsa's mother.

Solution

Let y be the current age of Elsa's mother.

$\therefore$ Elsa's current age is $\frac{y}{7}$.

After three years, her mother will be $(y + 3)$ years old, Elsa will be $(\frac{y}{7} + 3)$ years old.

$$\begin{aligned}
\therefore\quad y + 3 + (\frac{y}{7} + 3) &= 46\\
y + \frac{y}{7} &= 40\\
7y + y &= 280\\
8y &= 280\\
y &= 35
\end{aligned}$$

$\therefore$ The current age of Elsa's mother is 35.

Checking:
The current age of Elsa $= \frac{35}{7} = 5$
$\therefore$ After three years, the sum of the ages of Elsa and her mother
$= (35 + 3) + (5 + 3)$
$= 46$

Classwork 4.9

Solution

Lewis is 32 years younger than his father. Seven years ago, his age is $\frac{1}{5}$ of his father's. Find the current age of Lewis' father.

Additional Example 4.10

Example 4.10 Solving problems by using equations

Some books are evenly shared among a group of children. Each of them gets 39 books. If the number of children is reduced by 2, each of them will have 13 more books. How many books are there?

Solution

Let there be b books.

$\therefore$ Original number of children $= \frac{b}{39}$

If the number of children is reduced by 2,

the number of children $= \frac{b}{39} - 2$

the number of books each of them gets $= 39 + 13$

$$\therefore \quad (\frac{b}{39} - 2)(39 + 13) = b$$

$$52(\frac{b}{39} - 2) = b$$

$$52(\frac{b}{39}) - 52 \times 2 = b$$

$$\frac{4}{3}b - 104 = b$$

$$4b - 3 \times 104 = 3b$$

$$4b - 312 = 3b$$

$$b = 312$$

$\therefore$ There are 312 books.

Checking:

Original number of children $= \frac{312}{39} = 8$

If the number of children is reduced by 2,

the number of books each of them gets $= \frac{312}{8-2} = 52$

$\because \; 52 - 39 = 13$

$\therefore$ Each of them will have 13 more books.

Solution

Classwork 4.10

A sum of money is just enough to buy 12 pears. If the price of each pear is reduced by \$0.5, 3 more pears can be bought with the same amount of money. How much is the sum of money?

Skills Upgrading Corner 4.3

Solution

1. For a rectangle with the perimeter of 34 cm, its length is 3 cm longer than its width. Find the length and width of the rectangle.

Solution

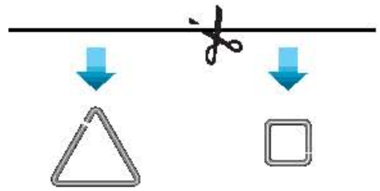
2. Grandfather is 6 times as old as Keith. 9 years ago, the difference of their ages is 60. Find grandfather's current age.

Solution

3. A sum of money is just enough to buy 35 vases. If the price of each vase increases by \$120, 10 less vases can be bought with the same amount of money. Find the original price of each vase.

Solution

4. A wire of 40 cm long is cut into two parts. The longer wire is bent to form an equilateral triangle. The shorter one is bent to form a square. If the length of each side of the triangle is 4 cm longer than that of the square, find the length of the longer wire.



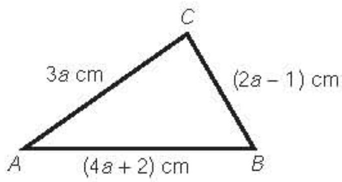
Exercise 4C

Level 1

1. Mr. and Mrs. Cheung get on a bus with their children. The bus fare for an adult is \$3.6, while for a child is half of an adult's fare. If they have to pay \$12.6 altogether, set up an equation for finding the number of children with them.

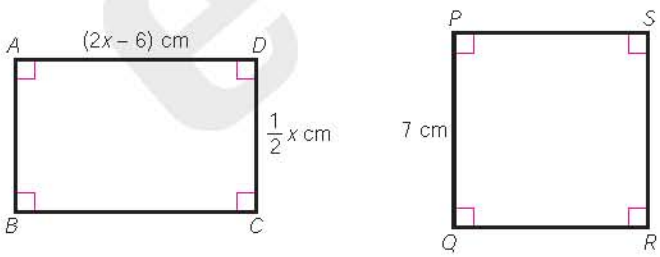
2. After 16 years, Kenneth will be 2 times as old as he was 12 years ago. Set up an equation for finding his current age.

3. In the figure, the perimeter of ΔABC is 91 cm. Find a.



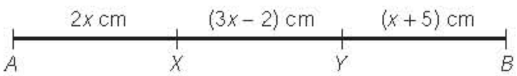
4. Scot has $\$(3x+2)$. Anson has $\$(4x-1)$. Karen has $\$(x+20)$. If they have \$69 in total, find x.

5. In the figure, if the perimeters of rectangle $ABCD$ and square $PQRS$ are the same, find the value of x.



6. Bobby buys x pieces of fruit tarts and eats $\frac{1}{4}$ of them. His brothers and sisters eat 7 pieces in total and there are 2 pieces left. Find the value of x.

7. In the figure, the length of AB is 27 cm. Find the length of AX.



8. 90 minus 6 times a number is 36. Find the number.

9. After subtracting 8 from a number, the difference multiplied by 9 is 108. Find the number.

10. Winnie and Lily have a total amount of \$40. If Winnie's amount is 4 times as much as Lily's, find Lily's amount.

11. Simon is 32 years older than his son. After 6 years, the sum of their ages will be 50. Find the current age of Simon's son.

12. The total price of 24 books is \$405. Some of them are sold at \$15 each and the others at \$20 each. Find the number of books that are sold at \$15 each.

Level 2

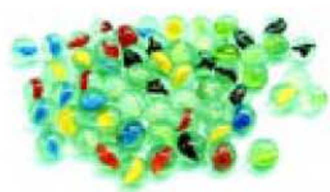
13. York has \$72. Sally has \$12. In order to let Sally have 3 times the amount of York, how much should York give Sally?

14. A sum of money is planned to be evenly shared among 10 persons. Due to the absence of 2 persons, each person present can get \$30 more. Find the sum of money.

15. Carl has \$9 more than David does. Hilbert has $\frac{1}{3}$ of the amount of Carl. If they have a total amount of \$61, how much does each of them have?

16. There are 42 students in S1A. The number of boys is $\frac{4}{3}$ times that of girls. Find the respective number of boys and girls there.

17. 72 marbles are divided into two groups. 7 times the number of marbles in the smaller group is equal to 5 times that in the larger group. Find the number of marbles in the smaller group.



18. The total age of Hebrew and his son is 51. 3 years ago, Hebrew is 8 times as old as his son. Find the current age of Hebrew's son.

19. Concert tickets in \$400, \$200 and \$100 each are available for sale. The number of \$200 tickets sold is 4 times as many as that of \$400 tickets. The number of \$100 tickets sold is 380 more than that of \$200 tickets. If the total income from the sale of tickets is \$86 000, how many \$400 tickets are sold?

20. **(a)** The following shows a sequence of figures. Draw the next three figures.

1st figure   2nd figure   3rd figure

(b) Complete the table.

Order of figure	1	2	3	4	5	6
Number of dots						

(c) Find the number of dots in the nth figure.

(d) If there are 200 dots in the nth figure, find the value of n.

Chapter Summary

Chapter Summary

A. Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

Column A		Column B
1. Unknown •	• **(a)**	An equality with an unknown.
2. Equation •	• **(b)**	A letter representing the unknown value in an equation.
3. Solution •	• **(c)**	A value satisfying the equation when it is substituted into the equation.

B. Fact to Remember

The following are the steps to solve a problem by using an equation:

1. Identify all unknown values in the problem.
2. Introduce an unknown to represent one of the unknown values.
3. Express the remaining unknown values in terms of the unknown introduced.
4. Set up an equation according to the problem.
5. Solve the equation.
6. Check the answer.
7. Answer the problem in words.

Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

1. **(a)** $x = 7$ is / is not a solution of equation $15-2x=7$. 4.1

(b) Solve $\frac{x}{3}+4=9$.

2. Solve the following equations. 4.2A

(a) $-6x+5x=3$ **(b)** $6x-8=14x+8$

3. Solve the following equations. 4.2B

(a) $-2(x-9)=2x-6$ **(b)** $7(x+9)-12(x+2)=-11$

4. Solve the following equations. 4.2C

(a) $\frac{12-x}{2}=\frac{2x+11}{3}$ **(b)** $\frac{2x-10}{3}-9x=5$

5. Calvin's marbles is half of Daisy's. If Daisy gives 14 marbles to Calvin, they will have the same number of marbles. Let Calvin have x marbles originally. 4.3B

(a) Set up an equation of x.

Equation: ______________________________

(b) How many marbles does each of them have originally?

Revision Exercise 4

Level 1

Solve the following equations. (**1** – **4**)

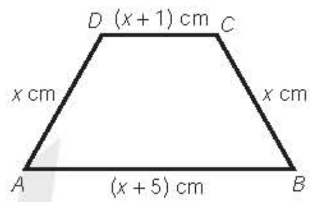
1. **(a)** $3x-5x=10$ **(b)** $-8x-9=4x+3$ **(c)** $14-11x=15x-12$

2. **(a)** $y+6=2(y-5)$ **(b)** $4(3-y)-5(6-y)=0$ **(c)** $2(y+3)+3(2y-1)+13=0$

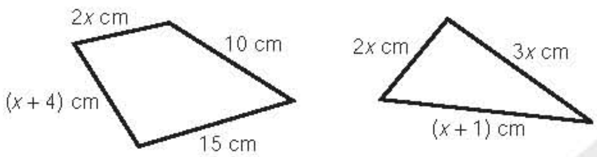
3. **(a)** $2(2x+3)=-18$ **(b)** $\dfrac{3x+2}{5}=1$ **(c)** $\dfrac{2}{3}(4x-1)=10$

4. **(a)** $\dfrac{y-12}{7}=20-y$ **(b)** $\dfrac{12-y}{2}=\dfrac{y+16}{5}$ **(c)** $\dfrac{y-5}{2}-\dfrac{6y}{7}=0$

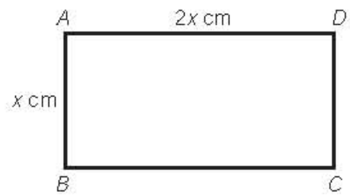
5. In the figure, the perimeter of trapezium $ABCD$ is 46 cm. Find the value of x.



6. In the figure, the perimeter of the quadrilateral is 2 times of the triangle's. Find the value of x.



7. In the figure, the perimeter of rectangle $ABCD$ is 60 cm. Find the area of the rectangle.



8. A number plus 13 is equal to 3 times the result of the number minus 11. Find the number.

9. For two consecutive even numbers, 3 times the larger number is 22 more than 2 times the smaller number. Find the two numbers.

10. The total weight of 130 packs of sugar is 134 kg. Some of them weigh 0.5 kg each and the others weigh 2 kg each. How many packs of sugar weigh 0.5 kg each?

11. *Model house* A is \$6 cheaper than model house B. The total price of 5 models house A and 3 models house B is \$210. Find the price of a model house A.

model house 模型屋

Level 2

Solve the following equations. (**12** – **14**)

12. **(a)** $6[2x-3(x+1)]=2x-10$ **(b)** $2[x-(2-3x)]=13x+4$

(c) $2[(3x-4)+2x]=x+10$ **(d)** $5[2(x-3.2)+x]=x-4$

13. **(a)** $\frac{12-2x}{5}=\frac{x+16}{3}$ **(b)** $\frac{4x+8}{2}=\frac{5x}{3}+8$

(c) $\frac{x+1}{4}+\frac{x+2}{3}=\frac{3}{2}$ **(d)** $\frac{5x-9}{3}-\frac{10-x}{2}=x-1$

14. **(a)** $(x-4)(x)+(2x-1)=x^2-(4-x)$ **(b)** $2(2x+1)(x)=3x^2+x(x-1)$

15. When you add a number to both sides of a linear equation in one unknown, then multiply both sides of the equation by the same number, the solution to the equation can be obtained. Set up an equation to satisfy the above.

16. The root of the equation $ax+b=cx+d$ is 2, where a, b, c and d are all non-zero integers. Find a set of values of a, b, c and d.

17. Larry is 4 times as old as Tim. After 8 years, the sum of their ages will be 56. Find the respective current ages of Larry and Tim.

18. A wire of 53 cm long is cut into two parts. The longer wire is bent to form an equilateral triangle. The shorter one is bent to form a square. If the length of each side of the square is 6 cm shorter than that of the triangle, find the respective lengths of the longer and shorter wires.

19. Howard drives at a speed of x km/h for the first 2 hours. Then the speed increases by 15 km/h for another 3-hour drive. If the distance of the whole journey travelled by Howard is 295 km, find the value of x.

[Hint: Speed = $\frac{\text{Distance}}{\text{Time}}$]

20. Elle is 28 years younger than Wesley. Keith's current age is $\frac{1}{4}$ of Elle's. Kiki is 18 years older than Keith. If Kiki is 28 years old now, find the current age of Wesley.

21. Tom has a sum of money to buy 60 stamps. If the price of each stamp increases by \$0.3, 12 less stamps can be bought with the same amount of money. Find the price of each stamp after the price increase.

22. Mr. Wong is granted the *annual paid leave* by his company as follows:
7 days for the 1st year of service; additional 2 days for every year of service thereafter.

(a) Consider the annual paid leave of Mr. Wong for the first four years. Complete the following table.

	Annual paid leave (day)
1st year	7
2nd year	$7 + 2 = 7 +$ ______ $\times 2$
3rd year	$7 + 2 + 2 = 7 +$ ______ $\times 2$
4th year	$7 + 2 + 2 + 2 = 7 +$ ______ $\times 2$

(b) If Mr. Wong has been working n years for the company, how many days of annual paid leave is he granted?

(c) The maximum annual paid leave granted by the company is 25 days. In which year of service will Mr. Wong be granted the maximum days of annual paid leave?

MC Question

23. If $3(x-5)=5-x$, then $x=$

A. -5.
B. 2.5.
C. 3.5.
D. 5.

24. Which of the following is/are linear equation(s) in one unknown?

I. $4x+5=-3$
II. $3x+2=6x-4$
III. $4x^2=16$

A. I only
B. I and II only
C. I and III only
D. II and III only

25. The original price of a bag is $\$x$. After a price reduction, the bag is sold at $\frac{4}{5}$ of the original price and is \$24 cheaper. Set up an equation to find x.

A. $\frac{4x}{5}=24$
B. $x-\frac{4x}{5}=24$
C. $\frac{4x}{5}-x=24$
D. $x+\frac{4x}{5}=24$

26. Michael spends \$52 on \$1.4 stamps and \$3 stamps. The number of \$1.4 stamps bought is 12 more than that of \$3 stamps. Suppose he buys x pieces of \$1.4 stamps, which of the following equations can find the number of \$1.4 stamps bought?

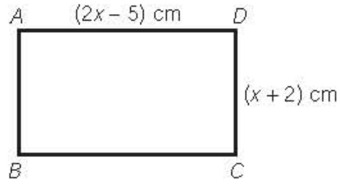
A. $1.4x+3x=52$
B. $1.4(x-12)+3x=52$
C. $1.4x+3x-12=0$
D. $1.4x+3(x-12)=52$

annual paid leave 有薪年假

27. The sum of two consecutive even numbers is 22. Their product is

A. 80.
B. 120.
C. 440.
D. 528.

28. In the figure, the perimeter of rectangle $ABCD$ is 54 cm. $CD =$

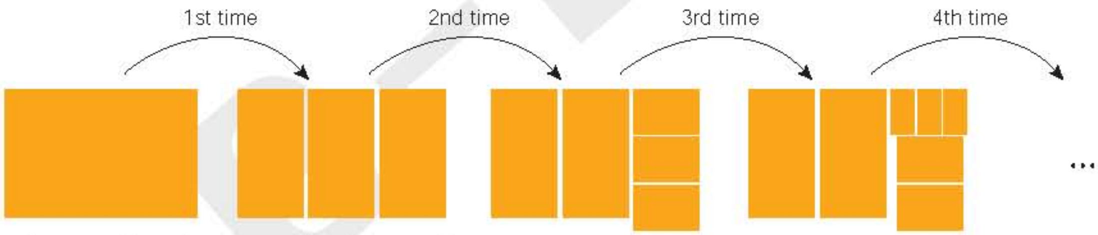


A. 10 cm.
B. 12 cm.
C. 15 cm.
D. 21 cm.

Problem-solving and Exploring

Hint for the Title Page Question

A sheet of paper is cut into 3 pieces. One of them is then cut into another 3 pieces, and the process goes on with the same method.



After cutting the paper for the 1st time,
number of pieces of paper $= 3 = 2(1) + 1$

After cutting the paper for the 2nd time,
number of pieces of paper $= 5 = 2(2) + 1$

After cutting the paper for the 3rd time,
number of pieces of paper $= 7 = 2(3) + 1$

(a) After cutting the paper for the nth time, how many pieces of paper are there?

(b) Hence, can we get exactly 30 pieces of paper by using the above method?

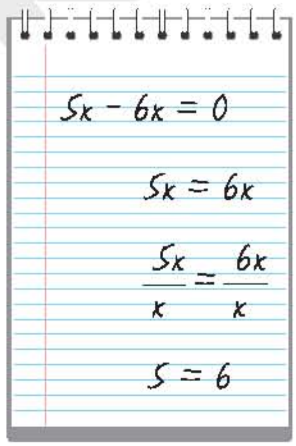
Additional Question

1. Two *squirrels* have collected *pine cones* for winter. One night, squirrel A evenly allocates the pine cones into two piles with one pine cone left unallocated. Squirrel A then takes one pile of pine cones and the one left unallocated.

   There comes squirrel B, it does the same by evenly allocating the remaining pile of pine cones into two piles with one pine cone left unallocated. Squirrel B also takes one pile of pine cones and the one left unallocated.

   The next day, they equally share the pile of pine cones left and each of them gets 25. How many pine cones does each of them get?

2. The following shows a solution in Oscar's homework. An unreasonable result of '5 = 6' is found. Can you point out the mistake for this?



squirrel 松鼠 pine cone 松果